Are Standard Solar Models Reliable?

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The sound speeds of solar models that include element diffusion agree with helioseismological measurements to a rms discrepancy of better than 0.2% throughout almost the entire sun. Models that do not include diffusion, or in which the interior of the sun is assumed to be significantly mixed, are effectively ruled out by helioseismology. Standard solar models predict the measured properties of the sun more accurately than is required for applications involving solar neutrinos.

For almost three decades, a discrepancy has existed between solar model predictions of neutrino fluxes and the rates observed in terrestrial experiments. In recent years, the combined results from four solar neutrino experiments have sharpened the discrepancy in ways that are independent of details of the solar models [1]. This development is of broad interest since a modest extension of standard electroweak theory, in which neutrinos have small masses and lepton flavor is not conserved, leads to results in excellent agreement with experiments [2].

Since the implications of a discrepancy with the standard electroweak model are of great importance, the question persists: Can the solar neutrino problems be "solved" (or at least alleviated) by changing the solar model? This question has led to a series of generally unsuccessful *ad hoc* "Non-Standard" solar models [3] in which large changes in the physics of the sun are hypothesized in order to lower the calculated rate of the ^{8}B neutrino flux. Over the past two decades, the most often hypothesized change is some form of mixing of the solar material that reduces the central temperature and therefore the important ^{8}B neutrino flux [4–9]. Previous arguments that extensive mixing does not occur are theoretical, including the fact that the required energy is five orders of magnitude larger than the total present rotational energy [3,9,10]. Most recently, Cumming and Haxton [11] proposed a flow of ^{3}He, characterized by three free parameters, designed to mix the sun in such a way as to minimize the discrepancy between solar neutrino observations and predictions. By adjusting the parameters, these authors are able to reduce the calculated ^{7}Be flux more than the ^{8}B flux, a result not achieved in previous Non-Standard solar models.

The diagnostic power of helioseismology [12] has been improved recently through the development by Tomczyk *et al.* [13] of an instrument that measures with the same equipment the low- and intermediate-degree mode frequencies. By providing a consistent set of frequencies for the lowest-degree modes, which penetrate to the greatest depth in the sun, these data constrain the properties of the solar core more tightly than earlier measurements.

In this letter, we compare the solar sound speed c inferred from the first year of data [14] with sound speeds computed from standard solar models used to predict solar neutrino fluxes and find a rms agreement better than 0.2% over essentially the entire sun, with *no* adjustment of parameters. Since the deep solar interior behaves essentially as a fully ionized perfect gas, $c^2 \propto T/\mu$ where T is temperature and μ is mean molecular weight; thus even tiny fractional errors in the model values of T or μ would produce measurable discrepancies in the precisely determined helioseismological sound speed

$$\frac{\delta c}{c} \simeq \frac{1}{2}\left(\frac{\delta T}{T} - \frac{\delta \mu}{\mu}\right) . \tag{1}$$

This remarkable agreement between standard predictions and helioseismological observations rules out solar models with temperature or mean molecular weight profiles that differ significantly from standard profiles. The helioseismological data essentially rule out solar models in which deep mixing has occured (cf. [15]) and argue against unmixed models in which the subtle effect of particle diffusion–selective sinking of heavier species in the sun's gravitational field–is not included.

Figure 1 compares the sound speeds computed from three different solar models with the values inferred [12,14] from the helioseismological measurements. The 1995 standard model of Bahcall and Pinsonneault (BP) [16], which includes helium and heavy element diffusion, is represented by the dotted line; the corresponding BP model without diffusion is represented by the dashed line. The dark line represents the best solar model which in-

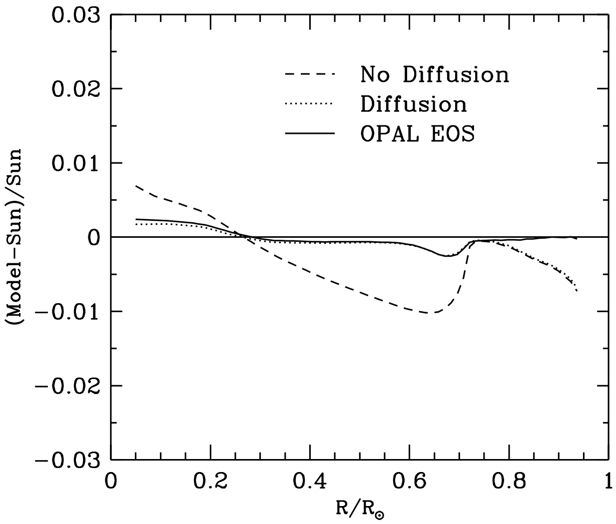

FIG. 1. Comparison of sound speeds predicted by different standard solar models with the sound speeds measured by helioseismology. There are no free parameters in the models; the microphysics is successively improved by first including diffusion and then by using a more comprehensive equation of state. The figure shows the fractional difference, $\delta c/c$, between the predicted model sound speed and the measured [12,14] solar values as a function of radial position in the sun ($R_\odot$ is the solar radius). The dashed line refers to a model [16] in which diffusion is neglected and the dotted line was computed from a model [16] in which helium and heavy element diffusion are included. The dark line represents a model which includes recent improvements in the OPAL equation of state and opacities [17,18].

cludes recent improvements [17,18] in the OPAL equation of state and opacities, as well as helium and heavy element diffusion. For the OPAL EOS model, the rms discrepancy between predicted and measured sound speeds is 0.1% (which may be due partly to systematic uncertainties in the data analysis).

In the outer parts of the sun, in the convective region between $0.7R_\odot$ to $0.95R_\odot$ (where the measurements end), the No Diffusion and the 1995 Diffusion model have discrepancies as large as 0.5% (see Figure 1). The model with the Livermore equation of state [18], OPAL EOS, fits the observations remarkably well in this region. We conclude, in agreement with the work of other authors [19], that the OPAL (Livermore National Laboratory) equation of state provides a significant improvement in the description of the outer regions of the sun.

The agreement between standard models and solar observations is independent of the finer details of the solar model. The standard model of Christensen-Dalsgaard *et al.* [20], which is derived from an independent computer code with different descriptions of the microphysics, predicts solar sound speeds that agree everywhere with the measured speeds to better than 0.2%.

Figure 1 shows that the discrepancies with the No Diffusion model are as large as 1%. The mean squared discrepancy for the No Diffusion model is 22 times larger than for the best model with diffusion, OPAL EOS. If one supposed optimistically that the No Diffusion model were correct, one would have to explain why the diffusion model fits the data so much better. On the basis of Figure 1, we conclude that otherwise standard solar models that do not include diffusion, such as the model of Turck-Chièze and Lopez [21], are inconsistent with helioseismological observations. This conclusion is consistent with earlier inferences based upon comparisons with less complete helioseismological data [12,22,15], including the fact that the present-day surface helium abundance in a standard solar model agrees with observations only if diffusion is included [16].

Equation 1 and Figure 1 imply that any changes $\delta T/T$ from the standard model values of temperature must be almost exactly canceled by changes $\delta\mu/\mu$ in mean molecular weight. In the standard model, T and μ vary, respectively, by a factor of 53 and 43% over the entire range for which c has been measured and by 1.9 and 39% over the energy producing region. It would be a remarkable coincidence if nature chose T and μ profiles that individually differ markedly from the standard model but have the same ratio T/μ. Thus we expect that the fractional differences between the solar and the model temperature, $\delta T/T$, or mean molecular weights, $\delta\mu/\mu$, are of similar magnitude to $\delta c^2/c^2$, i.e. (using the larger rms error, 0.002, for the solar interior),

$$|\delta T/T|,\ |\delta\mu/\mu|\ \lesssim\ 0.004. \qquad (2)$$

How significant for solar neutrino studies is the agreement between observation and prediction that is shown in Figure 1? The calculated neutrino fluxes depend upon the central temperature of the solar model approximately as a power of the temperature, Flux $\propto T^n$, where for standard models the exponent n varies from $n \sim -1.1$ for the $p-p$ neutrinos to $n \sim +24$ for the ^{8}B neutrinos [23]. Similar temperature scalings are found for non-standard solar models [24]. Thus, maximum temperature differences of $\sim 0.2\%$ would produce changes in the different neutrino fluxes of several percent or less, much less than required [1] to ameliorate the solar neutrino problems.

Figure 2 shows that the "mixed" model of Cummings and Haxton (CH) [11] (illustrated in their Figure 1) is grossly inconsistent with the observed helioseismological measurements. The vertical scale of Figure 2 had to be expanded by a factor of 2.5 relative to Figure 1 in order to display the large discrepancies with observations for the mixed model. The discrepancies for the CH mixed model (dashed line in Figure 2) range from +8% to −5%. Since μ in a standard solar model decreases monotonically outward from the solar interior, the mixed model– with a constant value of μ– predicts too large values for the sound speed in the inner mixed region and too small values in the outer mixed region. The asymmetric form

of the discrepancies for the CH model is due to the competition between the assumed constant rescaling of the temperature in the BP No Diffusion model and the assumed mixing of the solar core (constant value of μ). We also show in Figure 2 the relatively tiny discrepancies found for the new standard model, OPAL EOS.

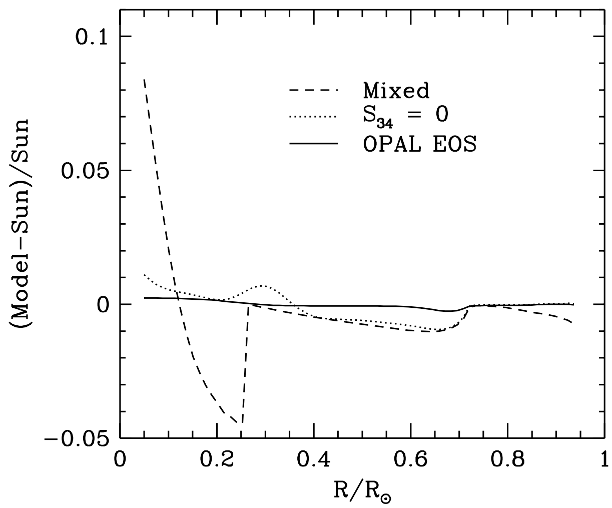

FIG. 2. Non-standard solar models compared with helioseismology. This figure is similar to Figure 1 except that the vertical scale is expanded. The dashed curve represents the sound speeds computed for the mixed solar model of Cumming and Haxton [11] with ^{3}He mixing. The dotted line represents the sound speed for a solar model computed with the rate of the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction set equal to zero. For comparison, we also include the results for the new standard model labeled OPAL EOS in Figure 1.

More generally, helioseismology rules out all solar models with large amounts of interior mixing, unless finely-tuned compensating changes in the temperature are made. The mean molecular weight in the standard solar model with diffusion varies monotonically from 0.86 in the deep interior to 0.62 at the outer region of nuclear fusion ($R = 0.25R\odot$) to 0.60 near the solar surface. Any mixing model will cause μ to be constant and equal to the average value in the mixed region. At the very least, the region in which nuclear fusion occurs must be mixed in order to affect significantly the calculated neutrino fluxes [3–7]. Unless almost precisely canceling temperature changes are assumed, solar models in which the nuclear burning region is mixed ($R \lesssim 0.25R_\odot$) will give maximum differences, δc, between the mixed and the standard model predictions, and hence between the mixed model predictions and the observations, of order

$$\frac{\delta c}{c} \;=\; \frac{1}{2}\left(\frac{\mu - <\mu>}{\mu}\right) \;\sim\; 7\% \text{ to } 10\%, \tag{3}$$

which is inconsistent with Figure 1.

Are the helioseismological measurements sensitive to the rates of the nuclear fusion reactions? In order to answer this question in its most extreme form, we have computed a model in which the cross section factor, S_{34}, for the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction is artificially set equal to zero. The neutrino fluxes computed from this unrealistic model have been used [3] to set a lower limit on the allowed rate of solar neutrinos in the gallium experiments if the solar luminosity is currently powered by nuclear fusion reactions. Figure 2 shows that although the maximum discrepancies ($\sim 1\%$) for the $S_{34} = 0$ model are much smaller than for mixed models, they are still large compared to the differences between the standard model and helioseismological measurements. The mean squared discrepancy for the $S_{34} = 0$ model is 19 times larger than for the standard OPAL EOS model. We conclude that the $S_{34} = 0$ model is not compatible with helioseismological observations (see also Ref. [25]).

Some nuclear parameters are important for solar neutrino experiments but have negligible effects on the computed solar model values of the sound speed. For example, we computed a standard solar model in which we artificially decreased by a factor of two the crucial cross section factor, S_{17}, for the rare $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reaction. The sound speeds computed for this radically different value of S_{17} differ by less than 1 part in 10^4 from the standard model values.

Finally, we comment on the effects of the recent improvements in opacity [17] and equation of state [18] on the predicted solar neutrino fluxes. Table I gives the neutrino fluxes computed for a series of three different standard solar models, all of which include helium and heavy element diffusion. The model labeled BP95 is from [16]; the models labeled New Opac and OPAL EOS include, respectively, the improved opacities discussed in [17] and the improved opacities plus the new OPAL equation of state discussed in [18].

TABLE I. Neutrino Fluxes for Solar Models with Diffusion. All fluxes, except for ^{8}B and ^{17}F, are given in units of 10^{10} per $\mathrm{cm}^{-2}\mathrm{s}^{-1}$ at the earth's surface. The ^{8}B and ^{17}F fluxes are in units of 10^6 per $\mathrm{cm}^{-2}\mathrm{s}^{-1}$.

Model	*pp*	*pep*	^{7}Be	^{8}B	^{13}N	^{15}O	^{17}F
BP95	5.91	0.014	0.515	6.62	0.062	0.055	6.48
New Opac	5.91	0.014	0.516	6.62	0.062	0.055	6.48
OPAL EOS	5.91	0.014	0.514	6.60	0.062	0.054	6.45

The neutrino fluxes computed with the improved opacity and equation of state differ from the previously published values [16] by amounts that are negligible in solar neutrino calculations. The predicted event rate, for all three models, is

$$\mathrm{Cl\ Rate} \;=\; 9.5^{+1.2}_{-1.4}\ \ \mathrm{SNU} \tag{4}$$

for the chlorine experiment and

$$\mathrm{Ga\ Rate} \;=\; 137^{+8}_{-7}\ \ \mathrm{SNU} \tag{5}$$

for the gallium experiments. The only noticeable change in the predicted event rates for the chlorine and the gallium experiment is a 2% larger event rate for chlorine, which is due to a small improvement [26] in the calculation of the neutrino absorption cross sections for ^{8}B.

We conclude that the recent improvements in opacity and equation of state do not significantly affect the calculated neutrino fluxes, although they do result in sound speeds near the solar surface that are closer to the measured helioseismological values (see Figure 1). The calculations of standard solar models lead to predicted sound speeds that agree closely with the measured helioseismological values. We cannot rule out with mathematical rigor the possibility [27] of constructing nonstandard models, consistent with quantum mechanics and with other stellar evolution observations, that are tuned to give the same sound speeds as the standard solar models. However, Ockham's razor suggests a strong preference for standard solar models.

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